Re-Opening After the Lockdown: Long-run Aggregate and Distributional Consequences of COVID-19*

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Abstract

Covid-19 has dealt a devastating blow to productivity and economic growth. We employ a general equilibrium framework with heterogeneous agents to identify the tradeoffs involved in restoring the economy to its pre-Covid-19 state. Several tradeoffs, both over time, and between key economic variables, are identified, with the feasible speed of successful reopening being constrained by the transmission of the infection. In particular, while more rapid opening up of the economy will reduce short-run aggregate output losses, it will cause larger long-run output losses, which potentially may be quite substantial if the opening is overly rapid and the virus is not eradicated. More rapid opening of the economy mitigates the increases in both long-run wealth and income inequality, thus highlighting a direct conflict between the adverse effects on aggregate output and its distributional consequences.

Keywords: Pandemic, containment policies, productivity, inequality, path dependence.

JEL Codes: E65, I14, I18, O11

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1. Introduction

COVID-19 is a crisis like no other that has trapped the global economy in a Great Lockdown. As countries have tried to manage the associated health risks that required locking down their economies, what has followed has been an unprecedented collapse in economic activity, combined with dramatic turbulence in financial and commodity markets. Several sectors have been hit particularly hard, including travel, the hospitality industry, and manufacturing, to name just a few. The strict containment measures while they have produced some positive results as they have broadly managed to curb the number of infections and related deaths, they also present a huge challenge for all countries, but especially for those developing economies unable to spend massive amounts on fiscal stimulus.

The distributional consequences of the pandemic may be even more severe and far more long-lasting than the growth and productivity impacts. The debate in both academic and policy circles on the magnitude of such impacts is grim and raises enormous concerns, especially for the part of the income distribution that includes the most vulnerable participants in the economy. The crisis by all estimates will also significantly increase poverty. According to the latest estimates by the World Bank, an additional 71 million people will be pushed into extreme poverty in 2020. The crisis will also push 176 million into broader poverty (living on less than $3.20 a day). And inequality may increase sharply, since the cost of lockdowns falls disproportionately on lower-income, informal workers: in low-income and lower-middle-income countries, whose earnings have decreased by 82 percent in the first month of the crisis. If inequality as measured by the Gini coefficient increases by one percent (not an unusual fluctuation even in normal times), the number of extreme poor would increase by almost 20 million. In addition, lockdowns are hampering food distribution, and spikes in food prices are further increasing hardship among poorer households; this year, an additional 130 million may suffer from acute hunger.

With countries preparing for the reopening of their economies, as part of the recovery phase of the crisis, they are actively experimenting using a variety of containment measures that confront policy makers with a very challenging calculus about saving lives and livelihoods. The aim of this paper is to shed new light on key mechanisms and implications of measures used in the process of reopening. The debate on how best to reopen the economies from the COVID-19 lockdown has an important long-run dimension that has received less attention in the literature. Our objective is to focus on this aspect. We show that while opening too fast may reduce short-run economic fallout, this comes at the cost of adverse long-run aggregate outcomes (output and consumption), while also introducing tradeoffs with respect to the impact on various inequality measures (wealth, health, and income).

This paper adopts the framework developed by Atolia, Chatterjee, and Turnovsky (2012; ACT thereafter) which employs a general equilibrium model with heterogeneous agents. The crucial mechanism generating the endogenous distribution of income is the relationship between agents' relative capital stock (wealth) and their respective allocation of time between work and leisure as the economy evolves following some structural change. In the long run, this relationship is positive, as wealthier agents who have a lower marginal utility of wealth increase their consumption of all goods, including leisure.[1] In the short run, however, this relationship is conditioned by the time path a given productivity change is expected to follow, and the differences in the consumption-smoothing motives it generates for rich and poor agents. A key feature of this labor allocation-relative wealth mechanism is that it introduces hysteresis in the dynamic adjustment characterizing the relative holdings of capital. Thus, a central insight of ACT is that the effects of a productivity change of a given magnitude on the long-run distributions of both wealth and income are crucially dependent upon the time path that the productivity change is assumed to follow. This is in sharp contrast to the dynamics of the aggregate economy, where the long-run equilibrium is independent of the transitional path.[2]

To examine the consequences of the process of opening, we integrate the ACT framework

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[1] To provide more context we should note that the ACT model, which emphasized the hysteresis aspect, builds on Turnovsky and García-Peñalosa (2008) which itself is an example of the "representative consumer theory of distribution" as coined by Caselli and Ventura (2000).

[2] This characteristic identified by ACT is in fact applies to any structural change introduced into their framework and so it offers a very natural approach to the current debate of "how fast to open" which is directly focusing on the choice of the appropriate transitional adjustment path.

into a recent two-sector Ramsey model of health Atolia, Papageorgiou, and Turnovsky (2019), extended to include the evolution of the COVID-19 virus, albeit in a stylized way. An important characteristic of the model is that the interaction between the speed with which the economy is opened and the spread of the virus – the key issue in the current debate – will determine the nature of the post-COVID-19 steady state. In contrast, to the basic ACT model, the interaction between the spread of the infection and the speed of opening up the economy may have long-run aggregate effects, as well as permanent distributional effects, depending upon the chosen speed. Moreover, the interaction between the speed of opening and the spread of the virus introduces various tradeoffs. Opening the economy more rapidly is likely to cause the virus to persist indefinitely, and while this may alleviate the short-run decline in aggregate economic activity, its permanence will exacerbate the long-run losses in production. In addition, more rapid opening with a faster transition will tend to mitigate the increase in long-run wealth and income inequality that the response to the pandemic generates. Our analysis also brings to the fore another source of inequality, namely health inequality, which is shown to be directly linked to wealth inequality.

Our numerical simulations suggest that, for what we view as a plausible rate of opening, the permanent effects on the economy are not inconsequential. For example, the loss in output after about four years is 2-3% at annual rates, although asymptotically it is much smaller, while the inequality may increase by 1.5%, which already noted is not trivial. Our results also suggest that since poorer economies may lack the infrastructure to open as rapidly as more developed economies, they are likely to suffer more adverse permanent distributional effects.

We should emphasize that, while much of the debate among policymakers identifies the increase in income inequality as an undesirable permanent consequence of the COVID-19 experience, the mechanism proposed in this paper is very different. Much of the debate attributes the inequality to small firms and businesses, temporarily closed during the pandemic, and being unable to recover. Our analysis generates the long-run inequality as a consequence of the intrinsic dynamics of the economy as it transitions in the process of reopening. This stems from the differential abilities/desires of individuals, having different endowments, to save (or dissave to smooth consumption), together with their corresponding/associated response of leisure.

The rest of the paper is organized as follows. Section 2 reviews the literature with particular focus on the distributional impact of COVID-19. Section 3 sets out the components of the model, while Section 4 describe the macroeconomic equilibrium. Section 5 specifies and derives the alternative inequality measures. Section 6 describes the calibration and the numerical simulations, while Section 7 draws the main conclusions.

2. Literature review

We begin by taking a brief look at the emerging literature on the impact of the Covid-19 pandemic on inequality and poverty. We also discuss work related to the modelling framework we use in the analysis and health as input in the production process.

As for the impact of Covid-19 on inequality, the majority of existing work focuses on advanced economies through the labor market channel. From the empirical side, cross-country work includes, Furceri, Loungani, Ostry, and Pizzuto (2020) who provide evidence on the impact of major epidemics from the past two decades on income distribution. Their results show past pandemics of this kind, even though much smaller in scale, have led to increases in the Gini coefficient, raised the income shares of higher income deciles, and lowered the employment-to-population ratio for those with basic education compared to those with higher education. Palomino, Rodriguez and Sebastian (2020) construct a Lockdown Working Ability index and estimate the potential wage loss under six lockdown scenarios across Europe. They find there would be substantial and uneven wage losses across the board; inequality within countries will worsen, as it will between countries although to a lesser extent. The impact will be felt particularly in the tail of the distribution with poverty likely to rise significantly.

In addition to cross-country analysis, there is a rapidly growth empirical literature focusing on selected advanced economies. Adams-Prassl, Boneva, Golin and Rauh (2020) using real-time survey evidence from the UK, US and Germany show that the labor market impacts of COVID-19 differ considerably across countries. Workers in alternative work arrangements and in occupations in which only a small share of tasks can be done from home, are more likely to have reduced their hours, lost their jobs and suffered falls in earnings. A key message of this paper is that less educated workers and women in particular are more likely to be affected by the crisis. Shibata (2020) compares distributional impacts of Covid-19 and those of the Global Financial Crisis (GFC) using the US Current Population Survey data. Empirical results show young and less educated workers have always been affected more in recessions, while women and Hispanics were more severely affected during current Pandemic Recession. And workers at low-income earnings suffered more than top income earners, suggesting a significant distributional impact of the two recessions. Chetty et al. (2020) build a platform tracking real-time economic activity across the US using anonymized data by private companies. In terms of inequality, they find high-income individuals' sharp consumption reduction in mid-March led to a surge in low-income unemployment claims in affluent areas. Also, children in high-income areas experience a temporary reduction in online learning but soon recover to baseline levels, whereas children in lower-income areas remain 50% below baseline levels persistently. Galasso (2020) exploits two real time surveys on the labor market after the lockdown in Italy and finds low-income individuals faced worse labor market outcomes and suffered higher psychological costs compared to highly educated and white collar workers.

According to Burgess and Sievertsen (2020), the global lockdown of education institutions is the cause of major and likely unequal interruption in students' learning but also disruptions in internal assessments and the cancellation of public assessments for qualifications or their replacement by an inferior alternative. The severe short-term disruption is felt by most families around the world already and are very likely to have a negative long-run impact in student's skill and productivity (see e.g., Burgess and Greaves; Carlsson et al. 2015; Lavy 2015). Depending on access to home schooling, and educational technology, education inequality is likely to increase in the future.

Beyond empirical evidence there is a growing literature focusing on alternative modeling frameworks to examine various impacts of the pandemic on inequality. Heathcote, Perri and

Violante (2020) model inequality in labor market during recessions. The authors build a structural model and find that deep recessions are likely to have long-lasting effects on the participation rates of low-skilled men and thus on earnings inequality. They then run several simulation exercises in the context of Covid-19 which strengthen their original findings. Glover, Heathcote, Krueger and Ríos-Rull (2020) build a model in which economic activity and disease progression are jointly determined. They study the optimal economic mitigation and redistribution policies of a utilitarian government and show that these policies interact and reflect a compromise between the strongly diverging preferred policy paths of different subgroups of the population. Kaplan, Moll and Violante (2020) expand their workhorse HANK model with liquid and illiquid assets to include an epidemiological SIR model and different types of occupations and sectors. Preliminary findings suggest that lockdowns hurt poor households disproportionately more and need to be in place for a very long time in order to be effective. These authors are currently working on more targeted policies, both in the health and economic fronts. Osotimehin and Popov (2020) model health and economic risks faced by different workers and how these risks cascade into other sectors through supply chains and demand linkages, exacerbating the unequal effects for certain sectors. They find that, in the US, the cascading effects account for about 25-30% of the exposure to both risks. Such effects increase the health risk faced by workers in the transportation and retail sectors as well as economic risks for workers in the textile and petroleum sectors.

So far, the literature review presented has been centered entirely around advanced economies. While much less is done by the profession on developing economies, there are some notable exceptions. Alon, Kim, Lagakos and VanVuren (2020) build a macroeconomic model with epidemiological dynamics including an informal sector and other characteristics more fitting to developing economies. The model predicts that blanket lockdowns are generally less effective in developing countries at reducing the welfare costs of the pandemic, and in saving lives per unit of lost GDP. The authors argue that age-specific lockdown policies may be even more potent in developing countries, saving more lives per unit of lost output than in advanced economies. Dasgupta and Murali (2020) integrate a standard epidemiological model within a general equilibrium framework and calibrate it to the Indian economy. Results show that different containment policies impose disproportionate economic costs on low-skill workers, thus worsening the already existing consumption inequality in the economy. Additionally, because low-skill workers do not have the luxury to work from home, the incidence of infections is also much higher. Lakner, Mahler, Negre and Prydz (2020) use model-based recursive partitioning to simulate scenarios for global poverty from 2019 to 2030 under various growth and inequality assumptions. They find reducing each country's Gini index by 1% per year has a larger impact on global poverty than increasing each country's annual growth 1% points above forecasts. Also, the pandemic may have driven over 60 million people into extreme poverty in 2020. Further, their analysis predicts that if the pandemic and associated economic crisis elevates Gini by 2% in all countries, it will push an additional 30 million people into extreme poverty.

Next, we briefly discuss work related to the modeling framework we use in the analysis. We also make reference to recent work that considers health as input in the production process. Atolia, Chatterjee and Turnovsky (2012), the paper which our analysis is based on, develops a heterogenous agent general equilibrium model where policy experiments can be numerically solved in a tractable manner. ACT is related to a growing body of research that exploits the fact that if the underlying utility function is homogeneous in its relevant arguments, the aggregate economy can be summarized by a representative agent, as a result of which aggregate behavior becomes independent of the economy's distributional characteristics. Rather, the distributions of income and wealth reflect the evolution of the aggregate economy as in Caselli and Ventura (2000), Kraay and Raddatz (2007), and Carroll and Young (2009). Awareness of this aggregation property dates back to Gorman (1953) and it has received renewed attention by researchers as the class of utility functions to which this aggregation applies includes the constant elasticity utility function that dominates contemporary growth theory.

In our analysis, ACT is used in conjunction with incorporating health in the production process in order to account for the epidemiological impact of Covid-19 shock. There exists a literature that focuses primarily on investigating the hypothesis that health status (measured as positively related to life expectancy, or inversely related to mortality or diseases) is a key determinant in explaining cross-country income differences via its direct or indirect effect on individuals' productivity and savings behavior; (see, e.g. Strauss and Thomas (1998); Deaton (2003); Lorentzen et al. (2008); Birchenall and Soares (2009); Chakraborty, et al. (2013); Bhattacharya and Chakraborty (2016); and Atolia, Papageorgiou, Turnovsky (2019). Similar to this literature, in our work health status is an important determinant of an agent's productivity.)[3] In summary, the empirical literature points to early evidence of very significant impact of the pandemic on inequality, with the most vulnerable being hit the hardest. This evidence provides ample motivation to theoretical contributions aiming at better understanding the mechanisms through which this shock will be impacting people, firms, and sectors differentially. Our paper fits into this strand of work, but it is different from the existing theoretical attempts in that it is more focused on assessing, in a tractable and intuitive manner, how the economy's transitional path and its ultimate steady state is impacted by the evolution of the pandemic shock.

3. The model

As noted, the model we employ is an adaption of Atolia, Papageorgiou, and Turnovsky (forthcoming), which introduces a health sector into a standard Ramsey growth model. The motivation for that project was the fact that in countries like the US, health accounts for around 18% of GDP and therefore surely merits serious analysis within the context of an advanced economy. Since the COVID-19 pandemic has hit developed economies, and most notably the US, it seems that this setup provides a reasonable framework within which to examine some of its macroeconomic and distributional consequences. The key modification we introduce is the assumption of heterogeneous agents, the source of the heterogeneity being due to their initial endowments of capital. While there are many potential sources of heterogeneity, there are at least two compelling reasons for focusing on this aspect. First, the seminal empirical evidence by

[3] There also exists a smaller set of theoretical contributions focusing on the effects of the decisions of individual agents to maximize, in addition to consumption, also their life expectancy (see, e.g., Blanchard, 1985; Ehrlich and Chuma 1990; and Allen and Chakraborty, 2018).

Piketty and his coauthors has focused on endowments and inheritance as a key underlying source of inequality.[4] Second, disparity in wealth seems most relevant in understanding the differential impact of the pandemic on the disparate members of the society. But, in part to maintain tractability and transparency, we abstract from one salient issue introduced in the previous paper, namely the impact of life expectancy on the rate of time preference. We justify this on the grounds that this pertains more to the long run, whereas the issues pertinent to the COVID-19 pandemic are more short-run in nature.[5]

3.1 Measures to control COVID-19 and their effect on productivity

Given the extremely infectious nature of the novel COVID-19 virus and the extreme health hazard it poses, through mechanisms little understood by the medical experts, governments all over the world responded by reducing human-to-human contact to prevent widespread loss of human life. As a result, human activity ground to a halt, with the effects on economic activity being particularly severe. In common parlance, economies were “locked down”.

We model the resulting overall decline in economic activity as a one-time discrete decline in total factor productivity of the final goods sector. In practice, the extent of decline would depend on the structure of production of an economy, its level of development, and the strength and scope of measures that were adopted.

3.2 Opening up the economy and the its health implications

As the economic costs of these measures were enormous, both in terms of output and wellbeing of the population, the governments started to slowly relax these measures to control spread of COVID-19. While this “opening up of the economy” restores the total factor productivity of the economy, by increasing the interaction between agents in the economy, it increases the transmission of the COVID-19 infection, thereby deteriorating the health status of workers, reducing their productivity, and adversely impacting the output of the economy. This

[4] See e.g. Piketty (2011), Piketty and Saez (2003), and Piketty and Zucman (2014).
[5] We also ignore population growth for a similar reason.

tradeoff, which has been generating a lively ongoing debate among politicians and public health officials, depends on the speed of the opening up of the economy, which is the focus of this paper.

In particular, let $A(t)$ denote the total factor productivity (TFP) at any arbitrary time *t,* with $\tilde{A}$ denoting the normal productivity level, prior to the onset of the COVID-19 crisis, and A_0 the productivity level immediately following the crisis, but prior to any opening up of the economy. The process of opening up the economy can then be conveniently described by the following differential equation

$$\dot{A}(t) = \theta\left(\tilde{A} - A(t)\right) \tag{1}$$

where θ is the speed with which this is occurring, and the target is the attainment of the pre-COVID-19 level of TFP. Thus, $A_0 < \tilde{A}$ and the difference (or ratio) of the two captures the impact of the initial anti-COVID-19 measures on the economy, which as mentioned earlier would depend on the strength and scope of those measures and the structure of the economy.

This paper takes these initial policy measures as given, and focuses on the choice of speed of opening up, θ, and its economic consequences. In particular, this policy choice of speed of normalization generates an important trade-off from an economic point of view between the rapidity with which TFP is restored to the original level (a plus for a higher θ) and the corresponding implications for health due to the spread of COVID-19 infections and resulting morbidity (a minus for a higher θ). The paper identifies broader adverse implications of deteriorating health, in particular highlighting its overall aggregate and distributional implications. These implications (for various measures of economic inequality) arise as the response of labor supply and saving (or dissaving to smooth consumption) of different households/agents in an economy depends on their initial wealth.

3.3 Formulation of dynamics of health and immunity

We begin with a stylized representation of the dynamics of the infection. Let $x(t)$ be the fraction of people infected with the virus at time *t*, and $y(t)$ be the fraction of the people who are still immune from infection due to past infection. Then, assuming a random meeting of people, we specify the rate of gross new infection as

$$g(\theta)x(t)\left(1-x(t)-y(t)\right) \tag{2}$$

where $x(t)(1-x(t)-y(t))$ measures the fraction of contact between an infected individual and an uninfected individual susceptible to infection, relative to all person-to-person contacts in the economy. The function $g(\theta)>0$, with $g'(\theta)>0$ measures the rate at which new infections arise from social interaction, together with the assumption that this will increase with the speed with which the economy is reopened, as it will increase personal contacts. We assume that the natural rate of recovery from the infection is a Poisson process with parameter κ in absence of any further exposure to infection. However, further exposure to infected people in the process of opening up the economy reduces the rate of recovery to $\kappa-\nu\theta$, where this reduction depends on the speed of opening up of the economy, and parameter $\nu>0$ controls the impact of speed on the recovery rate. Thus, the net rate of recovery is $(\kappa-\nu\theta)x(t)$ and the fraction of population infected evolves in accordance with

$$\dot{x}(t)=\left[g(\theta)\left(1-x(t)-y(t)\right)-(\kappa-\nu\theta)\right]x(t) \tag{3}$$

Our current understanding is that not all individuals who are infected are symptomatic and experience a deterioration in their health. Let ξ be the fraction of infected people who are symptomatic and suffer from adverse health consequences, when having active infection. Further, let their health decline in a relative sense to that of a healthy person to a level ψ . Then, the average health level of a household is given by

$$\Delta(t)\equiv 1-\xi x(t)+\xi x(t)\psi=1-\xi(1-\psi)x(t) \tag{4}$$

Finally, let χ be the fraction of those who recover that develop immunity to future infections and let ϖ be the rate of loss of immunity, then evolution of those who are immune is given by

$$\dot{y}(t)=\chi\left(\kappa-\nu\theta\right)x(t)-\varpi y(t) \tag{5}$$

To summarize: equations (3) and (5) describe the dynamics of those who currently infected (*x*) and immune from infection (*y*) and equation (4) describes the current impact of this dynamics on the average health (Δ) of a household, which enters our fairly standard macroeconomic model of the economy in the manner described in the subsequent sections. It is also clear that $x(t), y(t)$ being fractions are bounded between 0 and 1.

It is important to note that the dynamics of x and y are exogenous to the evolution of the macroeconomy, given the policy choice of speed of opening up, θ. Moreover, from the stationary solutions to (4) and (6), we see that there are two long-run interior equilibrium states of health:

$$\tilde{x}_1 = 0 = \tilde{y}_1 \tag{6a}$$

$$\tilde{x}_2 = \frac{\omega\left[g(\theta)+\nu\theta-\kappa\right]}{g(\theta)\left[\chi(\kappa-\nu\theta)+\varpi\right]}; \tilde{y}_2 = \frac{\chi(\kappa-\nu\theta)\left[g(\theta)+\nu\theta-\kappa\right]}{g(\theta)\left[\chi(\kappa-\nu\theta)+\varpi\right]} \tag{6b}$$

The equilibrium (6a) infection-free, with no individuals experiencing the virus, while in equilibrium (6b) the indicated fractions of agents will be experiencing the virus.

Which equilibrium emerges depends upon the speed with which the economy is opened up. To see this, consider the local dynamics of (3) and (5) around steady state, namely

$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} g(\theta)\left[1-2x-y\right]+\nu\theta-\kappa & -g(\theta)x \\ \chi(\kappa-\nu\theta) & -\varpi \end{pmatrix} \begin{pmatrix} x-\tilde{x} \\ y-\tilde{y} \end{pmatrix} \tag{7}$$

One can show from (7) that the economy will converge to the infection-free equilibrium if and only if $g(\theta)+\nu\theta<\kappa$, which imposes an upper bound on the rate at which the economy is opened up. Substituting (6b) it will converge to the equilibrium with infection if and only if $\kappa-g(\theta)<\nu\theta<\kappa+\varpi/\chi$. If, further, $\kappa+\varpi/\chi<\nu\theta$, the number of infections will simply diverge. The corresponding implications for the long-run health is obtained by substituting for $\tilde{x}$ from (6) into (4).

3.4 The production process

Production in the economy takes place in two sectors: a conventional final output sector, with each firm owned by a private individual, and a health sector, owned by the government as in Atolia, Papageorgiou, and Turnovsky (forthcoming). The representative firm in the final output sector produces in accordance with the conventional production function,

$$Y(t) = A(t)F[K(t), L(t), \Delta(t)\cdot h(t)] \quad F_K > 0, F_L > 0, F_h > 0 \tag{8a}$$

where K, L, Y denote aggregate stocks of capital, labor supply, and output and the production function is homogenous of degree one in *K* and *L*. In addition, following Bloom et al. (2004) who provide empirical evidence that labor productivity increases with the level of health, production also depends upon average health, $h(t)$, modified by the reduction in health due to the infection, as specified by equation (4). Producers take this as given, so that the state of health of workers serves as an externality insofar as the producers of final output are concerned.

The aggregate production function (8a) embodies the critical tradeoff between opening the economy and the likely adverse consequences for health and labor productivity alluded to earlier. The closing of the economy due to COVID-19 immediately reduces TFP to A_0 and the question is how fast to increase it to $\tilde{A}$, since the faster this occurs causes average health level Δ to decline, thereby offsetting (at least partially) the effect of increase in *A*. The firm chooses *K*, *L*, to maximize profit:

$$Y(t) = A(t)F[K(t), L(t), \Delta(t)\cdot h(t)] \;\; - r(t)K(t) - w(t)L(t) \tag{8b}$$

so that equilibrium factor returns are given by the usual marginal conditions

$$A(t)F_K[K(t), L(t), \Delta(t)\cdot h(t)] = r(t) \tag{9a}$$

$$A(t)F_L[K(t), L(t), \Delta(t)\cdot h(t)] = w(t) \tag{9b}$$

Health services are produced in accordance with the production function

$$h(m,e), \quad h_m > 0, \; h_e > 0 \tag{10a}$$

which is also homogeneous of degree one, in *m*, and *e*, where *m* is the aggregate health infrastructure/capital provided by the government, while *e* is the labor employed in the health sector. Thus (private) physical capital is specific to final goods production, while (public) health capital is specific to health services production. The health sector firm chooses employment, *e*, to maximize

$$ph(m,e) - we \tag{10b}$$

leading to the optimality condition

$$ph_e(m,e) = w, \tag{10c}$$

where p is the (relative) price of health. The homogeneity of the health production function means that the government earns profit, $p(h-h_e e)$, which contributes to its revenue.

3.5 Households

The economy is populated by a fixed number of households, represented as a continuum between 0 and 1, and each indexed by *i*. Households are identical in all respects except for their given initial endowments of capital, $K_{i,0}$, so that the average initial stock of capital in the economy is $K_0 = \int_0^1 K_{i,0} di$. At time *t*, with the accumulation of capital, the average per-capita amount of capital is correspondingly $K(t) = \int_0^1 K_i(t) di$, where $K_i(t)$ is the capital owned by household *i*. From a distributional perspective, we are interested in household *i*'s relative share of the total capital stock in the economy, $k_i(t)$, namely, $k_i(t) = K_i(t)/K(t)$. At all points of time, the mean of the distribution is normalized to unity, while the the initial (given) standard deviation of relative capital (the coefficient of variation of the level of capital) is $\sigma_{k,0}$.[6]

We now consider household *i*, which, like all others, is endowed with a unit of time that it can allocate to either leisure, l_i, or to work. The household derives utility from its consumption, C_i, leisure, l_i, and its health, h_i, which because of the infection caused by COVID-19 is reduced to $h_i\Delta$. Utility is thus represented by the following iso-elastic intertemporal utility function:

$$\max \int_0^\infty \frac{1}{\gamma}\left(C_i(t)l(t)_i^\eta (\Delta(t)\cdot h_i(t))^\omega\right)^\gamma e^{-\rho t}dt, \quad \text{with } -\infty < \gamma < 1, \eta > 0, \gamma\eta < 1 \tag{11a}$$

where $1/(1-\gamma)$ equals the intertemporal elasticity of substitution. This maximization is subject to the household's initial endowment of capital, $K_{i,0}$, together with its capital accumulation constraint

$$\dot{K}_i = \left[(1-\tau_k)r - \delta_k\right]K_i + (1-\tau_w)w(1-l_i) - C_i - p(1-s)h_i - T_i, \tag{11b}$$

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[6] We should emphasize that this formulation does not impose any particular distributional form, other than assuming the existence of a mean and an arbitrary measure of initial dispersion, $\sigma_{k,0}$. As will become clear later, the distributional dynamics of wealth and income we derive will reflect that of the arbitrary initial endowments, $\sigma_{k,0}$.

where r is return to capital, and δ_k is depreciation of capital, w is the wage rate, τ_k and τ_w are rates of capital and labor taxes, and T denotes the lump-sum tax. Equation (1b) also shows that the agent purchases health services at a price p, which may be subsidized by the government at the rate s. These health services are broadly defined to include medical services, pills, and even subscriptions to health clubs. For simplicity, we identify the purchase of these health services as being identical to health itself.[7] We also assume that the household may work either in the final output sector or in the health sector, with each sector paying the same wage.

Performing the optimization yields the following optimality conditions:

$$C_i(t)^{\gamma-1} l_i(t)^{\eta\gamma} (\Delta(t)\cdot h_i(t))^{\omega\gamma} = \lambda_i \tag{12a}$$

$$\eta C_i(t)^{\gamma} l_i(t)^{\eta\gamma-1} (\Delta(t)\cdot h_i(t))^{\omega\gamma} = w(1-\tau_w)\lambda_i \tag{12b}$$

$$\omega C_i(t)^{\gamma} l_i(t)^{\eta\gamma} (h_i(t))^{\omega\gamma-1} \Delta(t)^{\omega\gamma} = p\lambda_i(1-s) \tag{12c}$$

$$r(1-\tau_k)-\delta_k = \rho - \frac{\dot{\lambda}_i}{\lambda_i} \tag{12d}$$

together with the transversality condition $\lim_{t\to\infty} \lambda_i k e^{-z(t)} = 0$, where λ_i is the costate variable associated with the dynamic equation (11b).

From the optimality conditions (12) we immediately derive the following:

$$(\gamma-1))\frac{\dot{C}_i(t)}{C_i(t)} + \eta\gamma\frac{\dot{l}_i(t)}{l_i(t)} + \omega\gamma\left(\frac{\dot{h}_i(t)}{h_i(t)} + \frac{\dot{\Delta}(t)}{\Delta(t)}\right) = \frac{\dot{\lambda}_i}{\lambda_i} = \rho + \delta_K - r(t) \tag{13a}$$

$$\frac{\eta C_i(t)}{l_i(t)} = w(1-\tau_w); \quad \text{i.e} \frac{\dot{C}_i(t)}{C_i(t)} - \frac{\dot{l}_i(t)}{l_i(t)} = \frac{\dot{w}(t)}{w(t)}; \tag{13b}$$

$$\frac{\eta h_i(t)}{\omega l_i(t)} = \frac{w(1-\tau_w)}{p(1-s)}; \quad \text{i.e} \frac{\dot{h}_i(t)}{h_i(t)} - \frac{\dot{l}_i(t)}{l_i(t)} = \frac{\dot{w}(t)}{w(t)} - \frac{\dot{p}(t)}{p(t)} \tag{13c}$$

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[7] Several modifications to the formulation and interpretation of health services are possible. First, one could specify health as a more general positive concave function of resources devoted to health. Second, we could introduce health services as a stock rather than as a flow. In this respect, by relating h to the stock of public health capital (see 10a, above), it in fact retains much of the characteristics of a stock. Also, health costs are likely to be age-dependent. Since, our objective was to produce a simple canonical model, we refrained from introducing these modifications.

With all agents facing the same prices and having unimpeded equal access to all markets, equations (13a)-(13c) imply:

$$\frac{\dot{C}_i(t)}{C_i(t)} = \frac{\dot{C}(t)}{C(t)}; \ \frac{\dot{l}_i(t)}{l_i(t)} = \frac{\dot{l}(t)}{l(t)}; \ \frac{\dot{h}_i(t)}{h_i(t)} = \frac{\dot{h}(t)}{h(t)}; \ \frac{\dot{\lambda}_i(t)}{\lambda_i(t)} = \frac{\dot{\lambda}(t)}{\lambda(t)} \qquad \text{for all i} \qquad (14)$$

Thus all households choose the same growth rate for consumption, leisure, and health expenditures, implying further that average (aggregate) consumption, *C*, leisure, *l*, and health, *h* will also grow at the same common rate. Furthermore, equations (13b) and (13c) can be immediately aggregated to yield the equivalent relationships at the aggregate level

$$\frac{\eta C(t)}{l(t)} = w(1-\tau_w)\,; \qquad (13b')$$

$$\frac{\eta h(t)}{\omega l(t)} = \frac{w(1-\tau_w)}{p(1-s)} \qquad (13c')$$

3.6 Government

The government's budget constraint is

$$T = \dot{m} + \delta_m m + sph - \tau_k rk - \tau_w w(L+e) - p(h-h_e e)\,. \qquad (15)$$

According to (15) current government expenditures include its increase in health capital plus depreciation ($\dot{m} + \delta_m m$) and its subsidy to health expenditures (sph). Its revenues include the total tax collected ($\tau_k rk + \tau_w w(L+e)$), as well as profit earned by the health sector, $p(h-h_e e)$. To the extent that these items are not balanced it finances the difference with lump-sum taxes. We assume that the government devotes a fraction, *g*, of augment the aggregate stock of public health capital. Thus, we have

$$\dot{m} = gAF(K, L, h\Delta) - \delta_m m\,, \qquad (16)$$

which, using (7a)-(7b) enables us to rewrite the government budget constraint (15) in the form

$$T = gA(t)F(K,L,h) + sph - \tau_k A(t)F_k k - \tau_w A(t)F_L(L+e) - p(h-h_e e)\,. \qquad (15')$$

3.7 Market clearance

Labor is assumed to be both fully employed and enjoys free mobility across sectors:

$$L + e + l = 1. \tag{17}$$

Aggregating over the individual household's budget constraint, (11b), utilizing the government's budget constraint, (15'), recalling (9), and utilizing (10) yields the final goods market clearing condition

$$\dot{K} = (1-g)A(t)F(K, L, (\Delta \cdot h)) - C - \delta_k K, \tag{18}$$

4. Macroeconomic equilibrium

To appreciate how the opening of the economy impacts the macroeconomic equilibrium and ultimately the distribution of wealth and income, it is useful to begin by considering the steady state, which is obtained when $\dot{K} = 0$ and $\dot{\lambda}_i / \lambda = 0$.

4.1 Steady-state equilibrium

Setting $\dot{K} = 0 = \dot{\lambda}_i / \lambda_i$ in (18) and (12d) respectively, recalling equilibrium factor rates of return relationships (9a) and (9b), and using (10c) and (13c') to eliminate *p*, the steady state, denoted by ~ can be reduced to the following 8 equations:

$$\tilde{Y} = \tilde{A}F[\tilde{K}, \tilde{L}, \dot{h} \cdot \tilde{\Delta}] \tag{19a}$$

$$(1-g)\tilde{A}(t)F(\tilde{K}, \tilde{L}, \tilde{\Delta} \cdot \tilde{h}) - \tilde{C} - \delta_k \tilde{K} = 0 \tag{19b}$$

$$(1-\tau_k)\tilde{A}(t)F_K(\tilde{K}, \tilde{L}, \tilde{\Delta} \cdot \tilde{h}) = \rho + \delta_K \tag{19c}$$

$$\eta \frac{\tilde{C}}{\tilde{l}} = (1-\tau_w)\tilde{A}(t)F_L(\tilde{K}, \tilde{L}, \tilde{\Delta} \cdot \tilde{h}) \tag{19d}$$

$$\tilde{h} = h(\tilde{m}, \tilde{e}) \tag{19e}$$

$$\frac{\eta}{\omega}\frac{\tilde{h}}{\tilde{l}} = h_e(\tilde{m},\tilde{e})\left(\frac{1-\tau_w}{1-s}\right) \tag{19f}$$

$$\tilde{L} + \tilde{e} + \tilde{l} = 1 \tag{19g}$$

$$g\tilde{Y} = \delta_m \tilde{m} \tag{19h}$$

These 8 equations determine the steady-state solutions for the 8 variables $\tilde{Y},\tilde{K},\tilde{L},\tilde{h},\tilde{l},\tilde{e},\tilde{C}$, and $\tilde{m}$ in terms of the policy instruments, (g,τ_k,τ_w) and various structural parameters, including the level of technology $\tilde{A}$. Once the steady-state variables in (19) have been determined, other variables, including the relative price of health, $\tilde{p}$, and the consequences for the government budget, $\tilde{T}$, immediately follow.

Two characteristics of (19) merit comment. First, the long-run aggregate equilibrium is independent of any distributional characteristics. This is a well known consequence of the "representative consumer theory of distribution" on which our analaysis is based; see caselli and Ventura (2000). The more pertinent observation in the present context is that it also depends upon $\tilde{\Delta}$, which captures the equilibrium loss in health, due to infections that depends upon the speed with which the economy is opened up after the onset of COVID-19. By considering the dynamics of the virus' infection and recovery from it as summarized by equations (4)-(6) we may state the following proposition.

Proposition 1: (i) If the economy is opened up sufficiently slowly so that the condition $g(\theta)+\nu\theta<\kappa$ is met, agents will recover their health to its Pre-COVID level so that $\tilde{x}_1=0$, $\tilde{\Delta}=1$, then the aggregate economy will revert to its pre-COVI-19 steady-state equilibrium will be independent of the speed, θ, with which it is opened up.

(ii) If the economy is opened up at the rate satisfying $\kappa-g(\theta)<\nu\theta<\kappa+\varpi/\chi$, households will experience a long-run decline in their health $\tilde{\Delta}=1-\xi(1-\psi)\tilde{x}_2$, where $\tilde{x}_2$ is given in (6b) and the post-COVID-19 steady-state macroeconomic equilibrium will depend upon the speed of opening up, θ.

(iii) If the speed of opening up is sufficiently fast that $\kappa + \varpi/\chi < \nu\theta$ holds, then the spread of the virus will prevent the attainment of any interior macroeconomic equilibrium steady-state. Instead, $x(t) \to 1, y(t) \to 0$ and $\Delta(t) \to 1 - \xi(1-\psi)$, with the macroeconomic equilibrium suffering a larger permanent loss.

4.2 Transitional macroeconomic dynamics

To derive the transitional dynamics of the aggregate economy it is convenient to restrict ourselves to the functional forms of the production functions that we shall utilize in our numerical simulations, namely, the Cobb-Douglas specifications

$$Y = AK^{\alpha}L^{1-\alpha}(\Delta \cdot h)^{\beta} \tag{20a}$$

$$h = Bm^{\varphi}e^{1-\varphi} \tag{20b}$$

In Appendix A.1, we show how the aggregate dynamics can be reduced to a system of six equations in the three *endogenously* evolving varaibles $(\dot{K}, \dot{m}, \dot{l})$ and the three *exogenously* variables pertaining to the technology ($\dot{A}$) and the infection ($\dot{x}, \dot{y}$). The formal structure of the linearized dynamics is

$$\begin{pmatrix} \dot{K} \\ \dot{m} \\ \dot{l} \\ \dot{A} \\ \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15}(\theta) & 0 \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25}(\theta) & 0 \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35}(\theta) & 0 \\ 0 & 0 & 0 & -\theta & 0 & 0 \\ 0 & 0 & 0 & 0 & a_{55}(\theta) & a_{56}(\theta) \\ 0 & 0 & 0 & 0 & a_{65}(\theta) & a_{66}(\theta) \end{pmatrix} \begin{pmatrix} K - \tilde{K} \\ m - \tilde{m} \\ l - \tilde{l} \\ A - \tilde{A} \\ x - \tilde{x} \\ y - \tilde{y} \end{pmatrix} \tag{21}$$

Setting the dynamics out in this way highlights the channels whereby the speed with which the level of productivity is restored impacts the transition of the economy. In addition to the direct effect, it has secondary channels through the evolution of the infections and how they impact the health, and ultimately the evolution of capital and other elements of the economy.

Equation (21) summarizes the aggregate dynamics in a form analogous to that reported in the Appendix to ACT. The 3 x 3 submatrix involving $\dot{K},\dot{m},\dot{l}$ summarizes the internal dynamics of the economy, given the exogenous factors, the level of technology and the state of infections. Under weak conditions it is a saddlepoint, with $K(t)$ and $m(t)$ evolving gradually from their respective initial conditions, K_0, and m_0, while $l(t)$ is free to respond instaneously to new information. The improvement in productivity evolves gradually as the economy opens following the pandemic, and interacts with the evolving infections in impacting the evolution of the economy. In the absence of infections, the economy will ultimately revert to the pre COVID-19 steady-state, independent of θ, although the speed will affect the transitory path. The same continues to apply in the presence of infections, provided θ the speed of satisifies $g(\theta)+\nu\theta<\kappa$. If θ lies in the range $\kappa-g(\theta)<\nu\theta<\kappa+\varpi/\chi$, the aggregate economy will converge to the steady state characterized by case (ii) of Proportion 1, with the rate of opening up having a permanent impact on the aggregate economy. Finally, if θ exceeds this latter range, the rate of infection will be so intense that the aggregate economy will converge to the steady-state equilibrium that corresponds to setting $\tilde{\Delta}=1-\xi(1-\psi)$ in (19).

5. Distributional dynamics

To determine the evolution of the distributional variables in the economy we must consider $K_i(t)$ and $l_i(t)$. To do this we first recall the individual household's accumulation equation (11b). Substituting the optimality conditions (13b) and (13c) for households, together with the equilibrium wage rate we obtain

$$\dot{K}_i=\left[(1-\tau_k)r-\delta_k\right]K_i+(1-\tau_w)A(t)F_L\left[1-l_i\left(\frac{1+\eta+\omega}{\eta}\right)\right]-T_i \tag{22}$$

Summing (22) over the individual households we obtain

$$\dot{K}=\left[(1-\tau_k)r-\delta_k\right]K+(1-\tau_w)A(t)F_L\left[1-l\left(\frac{1+\eta+\omega}{\eta}\right)\right]-T \tag{23}$$

Thus, the evolution of the relative stock of capital owned by household *i*, $k_i(t) \equiv K_i(t)/K(t)$ evolves in accrdance with

$$\dot{k}_i = (1-\tau_w)\frac{A(t)F_L}{K(t)}\left\{1 - l_i\left(\frac{1+\eta+\omega}{\eta}\right) - \left[1 - l\left(\frac{1+\eta+\omega}{\eta}\right)\right]k_i\right\} \tag{24}$$

In deriving (24), in order to avoid arbitrary elements of distribution, we have assumed that lumpsum taxes are reallocated to individuals in proportion to their stock of capital $T_i/T = K_i/K$, which is perfectly consistent with the government's budget constraint. Recalling (14), $\dot{l}_i/l_i = \dot{l}/l$, it follows that $l_i = \upsilon_i l$, where υ_i is constant. Thus (24) may be rewritten as:

$$\dot{k}_i = (1-\tau_w)\frac{A(t)F_L}{K(t)}\left\{1 - \upsilon_i l\left(\frac{1+\eta+\omega}{\eta}\right) - \left[1 - l\left(\frac{1+\eta+\omega}{\eta}\right)\right]k_i\right\} \tag{24'}$$

from which we see that starting from an initial relative endowment, $k_{i,0}$, the evolution of the household's relative capital stock is driven by two factors: (i) the evolution of the aggregate quantities $K(t), l(t), A(t)$, as determined by (21) , and (ii) internally as determined by $k_i(t)$.

Setting $\dot{k}_i = 0$ in (24), we obtain the following long-run relationship between relative leisure and and relative capital

$$\tilde{l}_i - \tilde{l} = \left(\tilde{l} - \frac{\eta}{1+\eta+\omega}\right)(\tilde{k}_i - 1) \quad \text{for each } i \tag{25}$$

As shown in Appendix A.2, the coefficient of $(\tilde{k}_i - 1)$ is positive, implying that households having above average capital (wealth) enjoy above average leisure.[8]

While our simulations employ shooting algorithms to solve (24) for the time path of the relative stock of capital, in conjunction with the aggregate dynamics specified in (8a)-(8c), the intuition underlying the dynamic structure can be better understood by characterizing a linear approximation. To do this, we linearize (24') around the steady state. In the Appendix we show that the resulting bounded solution for the relative stock of capital is:

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[8] This is actually a strong implication, for which extensive empirical support exists as cited by Turnovsky and García-Peñalosa (2008).

$$k_i(t) - 1 = \varsigma(t)(\tilde{k}_i - 1) \tag{26}$$

where,

$$\varsigma(t) \equiv \left[1 + (1-\tau_w)\frac{\tilde{A}F_L}{\tilde{K}}\int_t^\infty \left(1 - \frac{l(\tau)}{\tilde{l}}\right)e^{-\pi(\tau - t)}d\tau\right].$$

and for convenience we define[9]

$$\pi \equiv (1-\tau_w)\frac{\tilde{A}F_L(\tilde{K},\tilde{L})}{\tilde{K}}\left[\tilde{l}\left(\frac{1+\eta+\omega}{\eta}\right) - 1\right]$$

Setting $t = 0$ in (25), we can solve for agent i's steady-state relative capital stock:

$$k_{i,0} - 1 = \varsigma(0)(\tilde{k}_i - 1) = \left(1 + (1-\tau_w)\frac{\tilde{A}F_L}{\tilde{K}}\int_0^\infty \left(1 - \frac{l(\tau)}{\tilde{l}}\right)e^{-\pi\tau}d\tau\right)(\tilde{k}_i - 1) \tag{26'}$$

where $k_{i,0}$ is given from the initial distribution of relative capital endowments.

5.1 Wealth inequality

Equations (25) and (26) characterize the evolution of relative capital. First, given the time path of the aggregate economy, in particular $l(\tau)$, and the distribution of initial capital endowments, (26) determines household i's steady-state relative holding of capital, $(\tilde{k}_i - 1)$. Once this is known, (25) then describes the time path of relative capital, which can be expressed in the convenient form[10]

$$k_i(t) - \tilde{k}_i = \left(\frac{\varsigma(t) - 1}{\varsigma(0) - 1}\right)(k_{i,0} - \tilde{k}_i) \tag{27}$$

Because of the linearity of (25)-(27), we can immediately transform these expressions into corresponding relationships for the standard deviation of the distribution of relative capital across agents, which serves as a convenient measure of wealth inequality:

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[9] In the absence of taxes and in the simpler one- sector model developed by Turnovsky and García-Peñalosa (2008) and utilized by Atolia, Chatterjee, and Turnovsky (2012) $\pi = \rho$, the rate of time preference.

[10] Note also that the constant $\upsilon_i = l_i/l$ can be determined from (25); it is given by $\upsilon_i = 1 + \left(1 - (1/\tilde{l})(\eta/(1+\eta))\right)(\tilde{k}_i - 1)$.

$$\sigma_k(t)-\tilde{\sigma}_k=\left(\frac{\varsigma(t)-1}{\varsigma(0)-1}\right)(\sigma_{k,0}-\tilde{\sigma}_k) \tag{28}$$

where, $\sigma_k(t)=\varsigma(t)\tilde{\sigma}_k$ and $\sigma_{k,0}=\varsigma(0)\tilde{\sigma}_k$.

In particular, writing

$$\tilde{\sigma}_k=\left(1+(1-\tau_w)\frac{\tilde{A}F_L}{\tilde{K}}\int_0^\infty\left(1-\frac{l(\tau)}{\tilde{l}}\right)e^{-\pi\tau}d\tau\right)^{-1}\sigma_{k,0} \tag{29}$$

highlights the mechanism whereby the rate of opening up the economy, θ, impacts the steady-state distribution of wealth. Assuming that θ is sufficiently slow that the economy converges to its pre-COVID-19 macroeconomic equilibrium, so that the long-run aggregate quantities are unchanged, the entire effect will be through its impact on the transitional path of leisure $\int_0^\infty\left(1-\frac{l(\tau)}{\tilde{l}}\right)e^{-\pi\tau}d\tau$, subsequent to its initial jump. For example, if during the transition $l(\tau)<\tilde{l}$, so that leisure approaches its long-run steady state from below, then $\varsigma(0)>1$ and wealth inequality will decline over time. As previous simulations of ACT have shown, in the absence of the infections, this is the case for a discrete productivity increase, where leisure increases (following an initial drop) and wealth inequality declines monotonically over time. On the other hand, a gradual productivity increase leads to an initial increase in leisure, taking it initially above its new (lower) steady-state level. But since the transitional path is U-shaped, eventually approaching $\tilde{l}$ from below, whether inequality rises or falls over time depends upon the extent to which $l(\tau)>\tilde{l}$ during the early phase of the adjustment. How this is affected by the infection, is not entirely apparent, and further light will be shed by the simulations we shall be reporting. But to the extent that having less healthy workforce with lower productivity reduces employment, $l(t)$ will tend to increase at each point of time, reducing $\varsigma(0)$ and causing $\tilde{\sigma}_k$ to increase. To the extent that this is the case, the speed with which the economy reopens will introduce a tradeoff between its impact on the long-run level of economic activity and its associated degree of inequality, as our simulations illustrate.

5.2 Income inequality

Defining household i's per capita income as $Y_i(t)=r(t)K_i(t)+w(t)\big(1-l_i(t)\big)$, and average economy-wide per capita income as $Y(t)=r(t)K(t)+w(t)\big(1-l(t)\big)$, we define relative income by $y_i(t)=Y_i(t)/Y(t)$. This leads to the following equation of motion for relative income:[11]

$$y_i(t)-1=\varphi(t)\big[k_i(t)-1\big] \tag{30}$$

where $\varphi(t)=1-(1-s(t))\left[1+\frac{l(t)}{1-l(t)}\left(1-\frac{\eta}{1+\eta+\omega}\right)\frac{1}{\varsigma(0)}\right]$

and $s(t)\equiv\frac{F_K(t)K(t)}{F_K(t)K(t)+F_L(t)(L(t)+e(t))}$

represents the share of capital in total income. Again, because of the linearity of (30) in $(k_i(t)-1)$, we can express the relationship between relative income and relative capital in terms of corresponding standard deviations of their respective distributions, namely

$$\sigma_y(t)=\varphi(t)\sigma_k(t) \tag{30'}$$

From (30') we see that the speed of opening the economy, together with the infections will have a permanent impact on income inequality.[12]

5.3 Health inequality

We can further show that the dynamics of opening the economy will have an effect on health inequality. To see this, we recall equation (13c) and (13c'), which together imply

$$\frac{h_i(t)}{h(t)}=\frac{l_i(t)}{l(t)}$$

From this equation we immediately infer

[11] See Turnovsky and García-Peñalosa (2008) for details regarding the derivation of the equations of motion for relative income and capital.

[12] The same comment applies to after-tax income inequality, which however, we do not consider.

$$\frac{h_i(t)-h(t)}{h(t)}=\frac{l_i(t)-l(t)}{l(t)}=\frac{\tilde{l}_i(t)-\tilde{l}(t)}{\tilde{l}(t)}=\left(1-\frac{\eta}{\tilde{l}(1+\eta+\omega)}\right)(\tilde{k}_i-1)$$

so that

$$\sigma_h(t)=\left(1-\frac{\eta}{\tilde{l}(1+\eta+\omega)}\right)\tilde{\sigma}_k \tag{31}$$

Thus, households experience a constant degree of health inequality, proportional to their long-run steady-state degree of wealth inequality.

As a final point we should emphasize that the source of long-run inequality being emphasized in this analysis, due to the response to the SARS-Cov-2 virus, is enirely different from that discussed in the media and among policy makers. In their case, it is typically because firms close down during the initial recession and are unable to recover and re-open. In our case, provided the recovery proceeds at the appropriate speed and the economy reverts to its pre-COVID-19 level of activity, there still will be inequality. This is because long-run wealth inequality depends upon the differential ability/desire of the individual households to accumulate assets while the economy is in transition.

6. Calibration strategy and numerical simulations

The analytical model set out in Sections 2 and 3 will be solved numerically, using the functional forms for utility, production, and social contact, specified in (11a), (20a), (20b) and in Table 1, together with the parameterization laid out in that table. We should emphasize that in contrast to the parameterization for the economic structure, which is for the most part well documented, the dynamics of the process of the spread of the COVID-19 infection is far less well known, including of course by the medical experts. Our strategy is to choose parameters of the process of spread of COVID-19 infection that generate a reasonably fast impact of the decision of opening up of the economy on transmission of infection and building of immunity consistent with observed contagious nature of the the SARS-Cov-2 virus. Our choice of $\kappa=5$, which specifies the natural rate of decay of the virus implies that with no personal interaction or intervention the rate of infection would be reduced by over 90% within 6 months. Figure 1 suggests that even for substantial values of θ the process runs its course in much less than a year. Our modelling of the spread of the COVID-19 infection is fairly flexible, which allows for a variety of assumptions about this infection process to be examined. This is an advantage since there is very little information about many important dimesions of this process. One such dimension is whether, SARS-Cov-2 virus will be completely eradicated or whether it will keep circulating forever like the flu virus. Our model allows for three scenarios: complete eradication, complete infection with no immunity, and middle case where there is persistent infection like the flu virus. We adopt this middle scenario as our benchmark, which is also consistent with recent comments by Dr. Anthony Fauci, Director, NIAID that this virus may keep circulating like the flu virus.[13] There is also concern raised in recent press reports about the duration of immunity gained from COVID-19 infection. We are able to examine the robustness of our results with respect to this source of uncertainty as well.

Turning to the economic parameters summarized in Table 1, their choice has been justifed at length in Atolia, Papageorgiou, and Turnovsky (2019). The parameters pertaining to final output, preferences, and choices of tax rates are extenively documented in the literature. The parameterization of the production function for health is less well documented, and the exponents have been chosen to yield a macroeconomic equilibrium in which: (i) the share of GDP due to health, and (ii) the total allocation of labor to the health sector approximates that of the United States.

6.1 Solution algorithm

As is well known, intertemporal models grounded in optimizing behavior typically yield saddle-point solutions, the exact numerical computation of which is often difficult. A typical procedure, therefore, is to derive linear approximations to the "true" dynamics, such as set out in (21). One alternative to deriving exact solutions for non-linear dynamic systems is to use some type of "shooting" algorithm (forward or reverse) to locate the path that lies on the stable

[13] https://www.cnbc.com/2020/07/22/dr-anthony-fauci-warns-the-coronavirus-wont-ever-be-totally-eradicated.html?__source=iosappshare%7Ccom.apple.UIKit.activity.Mail

manifold.[14] The choice between forward and reverse shooting depends on many factors, including the nature of the dynamic system, and the type of shock under consideration (Atolia and Buffie, 2009). Forward shooting computes the equilibrium path by searching over the initial values of the jump variables, whereas in reverse shooting the search is conducted over the terminal values of the state variables. For a dynamic model such as ours that is characterized by having a unit-root (zero-root in continuous time), so that the final steady-state values are not known, the forward shooting algorithm is the appropriate solution technique.

Accordingly, we solve the dynamics by employing a forward-shooting algorithm for unit-root systems developed by Atolia and Buffie (2011). As our complete dynamic system consists of $l, l_i, K, m, k_i, A, x, y$ and has two jump variables, we use the circle-search algorithm of Atolia and Buffie (2011) that underlies their *UnitRoot-Circle* program to obtain an exact solution to the dynamics of our unit-root system.[15] Since this algorithm allows solving for unit-root problems with two jump variables, we have the benefit of solving the complete dynamic system consisting of both the aggregate and the individual-level dynamics in a single step.[16]

6.2 Dynamics of infection

Figure 1 shows how the speed of opening, θ, interacts with the dynamics of infection, immunity, and health. We consider three different values of θ=1, 3, 11, with 3, which implies that after 6 months the economy is almost 80% open, as a plausible benchmark. As already noted, in this case, the virus keeps on circulating indefinitely, although, both infections and herd immunity reaches a steady level. Opening at the slower rate θ=1, the long-run impact of virus on economy is much smaller. At the other extreme, the scenario of immediate/fast opening, the focus of much of the current debate, is illustrated by θ=11 in Figure 1. In this case, the infection process becomes

[14] See Atolia and Buffie (2009) for other alternatives.
[15] Although we employ non-linear solution techniques for our numerical analysis, the first-order linearization procedure, albeit an approximation, is useful in guiding our intuition. Therefore, the linearized solutions for both the aggregate economy and the distributions are set out in the Appendix.
[16] Alternatively, because of the block-recursive structure one can solve the problem sequentially, first solving for the aggregate dynamics using a reverse-shooting procedure in Atolia and Buffie (2009) and then using this solution to solve for the "individual-level" dynamics using a unit-root forward-shooting algorithm.

explosive; everyone gets infected, which imposes a much larger health cost on the economy.

Table 3 summarizes the long-run consequences of the alternative speeds of opening for the key health characteristics. For $\theta =$ 1 or 3 the long-run fraction of infected individuals is relatively small, with a correspondingly small decline in the overall level of health. In contrast with very rapid opening everyone ends up being infected and the health level declines by 50%.

6.3 Dynamics of key aggregate variables

Figure 2 presents the implications of speed of opening for the aggregate economic variables. The way to think about the experiment illustrated here is the following. The closing of the economy due to COVID-19 occurs instantaneously at time 0, say. This is represented by the immediate drop of TFP, A, from its pre-COVID level of 1 to 0.8. The capital stock is fixed at that instant of time, but the reduction in productivity of reduces the demand for labor, which declines by around 18%. In addition, the loss of health contributes to a further loss of output, which overall on impact drops by 30% for the benchmark case of $\theta = 3$.

Immediately following the onset of COVID-19 the question of opening up the economy presents itself. As noted, in the absence of the virus, the economy would always eventually converge back to its pre-COVID steady state, irrespective of the speed of opening. However, for the benchmark value $\theta = 3$, the economy endures a permanent loss of health of 2.3%. As a result, there is some small permanent loss of aggregate activity relative to its pre-COVID-19 level.

The intuition for the dynamic response of the aggregate variables following the announcement of the intention to gradually reopen the economy generally follows that of ACT. The key element in explaining the dynamics is the following. On the one hand, the immediate closure of the economy leads to an instantaneous reduction in output. At the same time, with the anticipated gradual opening of the economy, agents know that over time output will eventually be restored to its previous level. Accordingly, permanent income, and therefore consumption upon which it is based, decline by less. Figure 2 illustrates that for the benchmark case, consumption immediately drops by around 8% rather than 30% as is the case for output. With the decline in output exceeding that of consumption, in order to maintain goods market equilibrium, investment must decline leading to a gradual decline of the capital stock. Over time, as the economy is gradually reopened in accordance with (1), the level of productivity and output gradually increases and the process of decline is gradually reversed. The economy gradually reverts back toward its pre-COVID-19 level of activity, although modified slightly by the persistence of the virus.

One striking feature of Figure 2 is that the persistence of the virus causes significant tradeoffs between the short-run and long-run effects associated with θ. Our simulations suggest that for the benchmark speed of opening, $\theta = 3$, the longer-run loss of annual output is around 1% after 4 years, and decline to just 0.21% asymptotically. Opening at the slower rate $\theta = 1$ causes the loss of output to be around 3% after 4 years, but eventually it is reduced to just 0.1%. In contrast, opening at the excessively rapid rate $\theta = 11$ reduces the output losses almost instantly but they never get below 4.5%, which is very significant and reflects the long-run costs of the adverse effects on health.

The transitional path of labor supply/employment generally mirrors that of output, although in all cases employment eventually returns to its pre-COVID-19 level. There is also some minor non-monotonicity associated with a slow opening, which is then reflected in the long-run wealth and income inequality. Faster opening also leads to a larger long-run decline in the aggregate capital stock, and while the long-run proportional response of capital reflects that of output (a direct consequence of the Cobb-Douglas production function), the short-run dynamics are different in several key respects. First, the decline in capital during the transition is inversely related to the speed of opening, θ, and second it is non-monotonic. This is because over time as A increases, after some point the increase in output generated is sufficient to accommodate an increasing investment. This reversal in the prior trend means that capital converges at a much slower rate to its new equilibrium than does output, implying that the economy sustains large losses of capital over an extended period of time.

Overall, Figure 2 highlights the short-run and long-run tradeoffs between the transition of the various aggregate variables as the speed of opening, θ, varies. An intermediate speed of θ=3 gives the best outcome, in terms of consumption over the short- and medium-run, which is arguably the time horizon of greatest relevance. However, it is also accompanied by largest decline in labor in the short run.

6.4 Dynamics of inequality

Figure 3 illustrates the dynamic evolution of wealth and income inequality. As is evident from the description of the formal model in Section 4, the driving force is the evolution of wealth inequality. In this regard, the crucial element is equation (29) which implies that the long-run steady-state degree of wealth inequality depends critically upon the transitional time path followed by leisure (and equivalently employment) following the opening up of the economy. For example, if $l(\tau)$ were to jump instantaneously to its steady-state $\tilde{l}$, (29) would reduce to $\tilde{\sigma}_k = \sigma_{k,0}$ and wealth inequality would remain unchanged. Intuitively, this relationship reflects the fact that it takes time to accumulate assets, and that wealth inequality results from the reality that given their diverse endowments, different agents find it optimal to save at different rates.

Comparing the long-run transitional time paths for the various inequality measures in Figure 3 suggests quite a contrast with the long-run responses of the aggregate measures illustrated in Figure 2. Focusing initially on wealth inequality we may note the following. In the benchmark case of $\theta = 3$, we see that wealth inequality converges quite rapidly to its new level following the opening of the economy, essentially reaching it after just a few months. Moreover, short-run wealth inequality increases by around 1.5%, before gradually declining to its long-run increase of 1.28%. While increases of this magnitude are not major, they are certainly not trivial either. With the Gini coefficient of wealth in the US being around 0.85 this would raise it to over 0.86.

The comparison of the two figures illustrates another tradeoff facing the choice of θ. On the one hand, increasing θ has been shown to increase the long-run losses in aggregate output. On the other hand, opening more rapidly reduces the increase in inequality. Thus, increasing θ from 1 to 3 and to 11, reduces the impact of opening on the increase in wealth inequality from 2.76% to 1.28% and to 0.45%. Increasing the speed of opening while it increases the long-run losses in aggregate output, it reduces wealth inequality. The intuition for this result is again provided by (29), the more rapidly the economy opens, the more rapidly the economy converges to its new steady state, the more rapidly leisure approaches its new steady state, the less time for the diverse savings/consumption-smoothing behavior of individuals to operate and consequently the less the impact on inequality.

To understand the consequences for income inequality we return to (30’) which expresses income inequality in terms of the current wealth inequality coupled with its impact on the relative income due to labor and capital. The fact that the choice of θ affects the long-run wealth inequality implies that the same applies to income inequality, although the time paths are very different. Since leisure eventually returns to its original steady state and with the production functions being Cobb-Douglas, $\varphi(t) \to \tilde{\varphi}$ a constant, so that the effect of θ on long-run income inequality mirrors its effect on wealth inequality. This is clearly evident by comparing the first two columns of Table 5. On impact, however, wealth inequality remains unchanged, and the short-run effect on income inequality is dominated by $\varphi(0)$, which drops sharply due to the sharp increase in $l(0)$, so that income inequality declines. This result is driven by the fact that richer households with more assets (capital) see not only their labor income (which varies less across households) but also their capital income fall sharply in the short run. Notice the path on income inequality in the short run closely mimics that of productivity. The short-run response of income inequality to θ is non-monotonic, with the greatest reduction occurring for the benchmark value of θ=3.

The message is fairly consistent. Inequality relative to its initial level – whether in wealth or income – rises unambiguously in the long-run when opening the economy is done more slowly. The wealth inequality increases because slower opening forces poorer households to use a large proportion of their assets to smooth consumption. This in turn translates into increased long-run income inequality.

Figure 3 also illustrates another, less discussed, but closely related measure of inequality, namely health inequality. One of the issues that the COVID-19 pandemic has laid bare is the wide disparity of health across the economy. Our model sets this out explicitly in equation (31), where it is shown that health inequality is strictly proportional to long-run wealth inequality. This result is intuitively appealing in view of the fact that health is a long-run investment and wealthier individuals allocate a larger proportion of their assets to health insurance and maintenance. While it is not subject to transitional dynamics, nevertheless to the extent that θ impacts the long-run wealth inequality it will impact health inequality as well, though to a lesser degree.

6.5 A robustness check

As noted earlier, there is a lot of uncertainty about how long the immunity from COVID-19 lasts. We examine the impact of this uncertainty on our results and find that our results for both aggregate and distributional dynamics are highly robust to a wide range of plausible levels of hazard rate of immunity, ϖ , which is set at 1/5 in the benchmark case. In particular, as illustrated in Figure 4, we find almost no difference in aggregate and distributional outcomes for values ranging from one-third as large (1/15) to as large as three times (3/5).

7. Concluding remarks

The COVID-19 virus has inflicted the greatest negative supply shock on the world economy in modern history. Not only is it far greater than the oil shocks of the 1970s, wreaking havoc on economies across the globe, attempts to restore the economies to their prior healthy states threaten the recurrence of the virus. In this paper we have focused on the tradeoffs between (i) the speed of reopening the economy, (ii) the spreading and persistence of the virus and (iii) the consequences for economic performance. Our paper has identified several aspects to this tradeoff, both over time, and between key economic variables, with the feasible speed of reopening being constrained by the state of the infection. This suggests that successful recovery of the economy will be a long and complex process, requiring careful coordination between the economic and epidemiological aspects.

First, we have shown that while more rapid opening up of the economy will reduce the short-run losses of output, it will be associated with larger long-run output losses, which in fact may become quite substantial if the opening is overly rapid and the virus is permanent. Second, different parts of the economy may be able to open at different rates. While output may mostly recover at a steady rate, the capital stock is likely to continue to decline for some initial period, slowing down its eventual rate of recovery and creating imbalances during early stages of the transition. Third, more rapid opening of the economy will reduce the increases in both long-run wealth and income inequality, thus highlighting a direct conflict between the adverse effects on the aggregate output losses and its more desirable impact on distribution across agents.

We should also emphasize that the impact of the speed of opening the economy on inequality is intrinsic to the macrodynamic system. It reflects the fact that during the transition, different agents are able to save at different rates, depending upon their resources. It is an entirely different source of inequality than that one hears about in policy discussions, which is associated with firms unable to reopen following the economic shutdown. Moreover, it is quite significant quantitatively, being potentially of the order of 1-2%.

Finally, we should note that our objective has been to identify and characterize the tradeoffs involved in the reopening of the economy. This naturally raises questions of policies that may facilitate this process, allowing the economy to reopen more rapidly while mitigating the adverse effects. There are two that immediately come to mind. First, there are policies associated with modifying social behavior in order to alleviate the transition of the virus. These relate to "social distancing" and the wearing of masks and are reflected in the function $g(\theta)$. The second is the role of fiscal policy and financial aid, the granting of $1200 to individuals, recently enacted by the US Congress being an example of the latter. This involves the government increasing its deficit at least temporarily, by borrowing and cannot be adequately addressed with the balanced budget assumption adopted here. It is straightforward to extend the model to include government debt and to address this issue, along with other forms of fiscal response in detail.

Table 1. Baseline parameter values

Parameters pertaining to virus and infection	
Personal interaction	$g_1\theta^{g_2}$ $g_1 = 10,\ g_2 = 2$
Natural rate of recovery	$\kappa = 5$
Impact of rate of opening on recovery	$\nu = 0.5$
Fraction of infected who are symptomatic	$\xi = 1$
Health of symptom. individuals	$\psi = 0.5$
Fraction of those that recover that develop immunity	$\chi = 1$
Rate of loss of immunity	$\varpi = 0.2$
Economic parameters	
Utility	$\gamma = -1.5$ (i.e. IES 0.4), $\eta = 1.5, \omega = 0.15$
Final Output	$A = 1,\ \alpha = 0.36,\ \beta = 0.05$
Health Production	$B = 0.4, \varphi = 0.55$
Rate of Time Preference	$\rho = 0.0396$
Government policy parameters	$g = 0.03,\ \tau_k = 0.276, \tau_w = 0.224,\ s = 0.64$
Depreciation rate	$\delta_k = 0.08, \delta_m = 0.04$

Table 2. Equilibrium ratios

Consumption-GDP ratio	0.79
Capital-output ratio	2.01
Allocation of time to leisure	0.659
Allocation of labor to final output production	0.272
Allocation of labor to health production	0.0291
Equilibrium rate of time discount	0.0396
Percentage of consumption devoted to health	6.5%
Public health as percentage of total health	62.4%
Total health as a percentage of GDP	15.8%

Table 3. Long-run Costs due to Infections under Alternative Opening up Scenarios

	Fraction Infected	Fraction Immune	Health level
$\theta = 1$	0.023	0.527	0.989
$\theta = 3$	0.053	0.909	0.973
$\theta = 11$	1.00	0.00	0.50

Table 4. Long-run Costs to Aggregate Quantities under Alternative Opening up Scenarios

	Aggregate Output	Aggregate Consumption	Aggregate Labor	Aggregate Capital	Aggregate Health(h*Delta)
$\theta = 1$	1-.999=.001	1-.999=.001	1-1=0	1-.999=.001	1-.9878=.0122
$\theta = 3$	1-.9979=.0021	1-.9979=.0021	1-1=0	1-.9979=.0021	1-.9729=.0271
$\theta = 11$	1-.9450=.0550	1-.9450=.0550	1-1=0	1-.9450=.0550	1-.9694=.0306

Table 5. Long-run Effects on Inequality under Alternative Opening up Scenarios

	Wealth	Income	Health
$\theta = 1$	1.02756	1.02865	1.02729
$\theta = 3$	1.01278	1.01315	1.01268
$\theta = 11$	1.00475	1.00272	1.00525

Appendix

A.1 Derivation of core aggregate dynamics

The aggregate dynamics consists of two components, (i) the internal dynamics, and (ii) the external dynamics associated with A and Δ. We employ the following functions:

$$Y = AK^{\alpha} L^{1-\alpha} (\Delta \cdot h)^{\beta}$$

$$h = Bm^{\varphi} e^{1-\varphi}$$

For these functions, the critical relationships reduce to the following:

$$Y = AK^{\alpha} L^{1-\alpha} (\Delta \cdot h)^{\beta} \tag{A.1a}$$

$$h = Bm^{\varphi} e^{1-\varphi} \tag{A.1b}$$

$$\frac{\eta C}{l} = (1-\alpha)(1-\tau_w)\frac{Y}{L} \tag{A.1c}$$

$$\frac{\eta}{\omega l} = \frac{(1-\tau_w)}{(1-s)} \frac{(1-\varphi)}{e} \tag{A.1d}$$

$$l + L + e = 1 \tag{A.1e}$$

These 5 equations can be solved for Y, C, h, e, L in terms of K, m, l, A, Δ of the form $Y = Y(K, m, l, A, \Delta)$ etc. The objective is to reduce the dynamics to an autonomous system in K, m, l, A, Δ, where $\Delta(t)$ is then expressed in terms of $x(t), y(t)$ in accordance with equations (3) and (4):

(i) The dynamics of $K(t)$ and $m(t)$ are both immediately obtained,

$$\dot{K} = (1-g)A(t)F[K, L(..), (\Delta(..) \cdot h(..))] - C(..) - \delta_k K \tag{A.2a}$$

$$\dot{m} = gY(K, m, l, A(t), \Delta(..)) - \delta_m m \tag{A.2b}$$

(ii) The dynamics of l are less direct and are obtained as follows. Taking the time derivatives of (A.1a)-(A.1f), yields

$$\frac{\dot{Y}}{Y} = \frac{\dot{A}}{A} + \alpha\frac{\dot{K}}{K} + (1-\alpha)\frac{\dot{L}}{L} + \beta\left(\frac{\dot{h}}{h} + \frac{\dot{\Delta}}{\Delta}\right) \tag{A.3a}$$

$$\frac{\dot{h}}{h} = \varphi\frac{\dot{m}}{m} + (1-\varphi)\frac{\dot{e}}{e} \tag{A.3b}$$

$$\frac{\dot{C}}{C}=\frac{\dot{l}}{l}+\frac{\dot{Y}}{Y}-\frac{\dot{L}}{L} \tag{A.3c}$$

$$\frac{\dot{e}}{e}=\frac{\dot{l}}{l} \tag{A.3d}$$

$$L\left(\frac{\dot{L}}{L}\right)+l\left(\frac{\dot{l}}{l}\right)+e\left(\frac{\dot{e}}{e}\right)=0 \qquad \text{(A.3e))}$$

Recalling (13) and (14) we also have

$$(\gamma-1)\frac{\dot{C}(t)}{C(t)}+\eta\gamma\frac{\dot{l}(t)}{l(t)}+\omega\gamma\left(\frac{\dot{h}(t)}{h(t)}+\frac{\dot{\Delta}(t)}{\Delta(t)}\right)=\rho+\delta_K-(1-\tau_k)\alpha\frac{Y(.)}{K} \tag{A.3f}$$

Using equations (A.3a)-(A.3f) one can eliminate $\dot{Y}/Y$, $\dot{C}/C$, $\dot{L}/L$, $\dot{e}/e, \dot{h}/h$ leaving a relationship relating $\dot{l}/l$ to $\dot{K}/K$, $\dot{m}/m$, $\dot{A}/A$, $\dot{\Delta}/\Delta$ and the functions obtained by solving for (A.1). Taken in conjunction with (A.2a) and (A.2b), together with the exogenous dynamics for A given in (1) and for Δ obtained from (3)-(5) can be solved. *K, A,* and Δ are all sluggish, evolving from initial states. K_0, A_0 would be given, the latter being the initial starting point following the closing down, while Δ_0 would be determined by the chosen rate of opening up, θ. At appears at this point is that the impact on permanent inequality stems from two gradual changes (i) A and (ii) Δ. It would seem that is that gradual opening up economically increases long-run inequality, while gradual recovery is likely to have the opposite effect.

A.2 Derivations of the expression $l>(\eta/(1+\eta+\omega)$ in (25)

This derivation imposes the weak constraints: (i) $\tau_k > g, \tau_w > g, s$. We begin with equation (18), which in steady state is

$$(1-g)\tilde{A}F(\tilde{K},\tilde{L},(\tilde{\Delta}\cdot\tilde{h}))-\tilde{C}-\delta_k\tilde{K}=0 \tag{A.4}$$

Using (i) the homogeneity of *F* in *K* and *L* and (ii) the steady state solution for $\tilde{C}$ from (19d). we may write:

$$\left[(1-g)\tilde{F}_K-\delta\right]\tilde{K}+\tilde{F}_L\left[(1-g)\tilde{L}-\frac{\tilde{l}}{\eta}(1-\tau_w)\right]=0 \tag{A.5}$$

Using the steady state optimality condition for capital, (19c), we find:

$$(1-g)\tilde{F}_K-\delta=\frac{(1-g)\rho+(\tau_k-g)}{1-\tau_k}>0\,.$$

Hence (A.5) implies

$$(1-g)\tilde{L}-\frac{\tilde{l}}{\eta}(1-\tau_w)<0 \tag{A.6}$$

Utilizing the steady-state labor allocation condition (19g), (A.6) may be rewritten as

$$(1-\tilde{l}-\tilde{e})<\frac{\tilde{l}}{\eta}\frac{(1-\tau_w)}{(1-g)}<\frac{\tilde{l}}{\eta} \tag{A.7}$$

Recalling the steady state optimality condition for labor to the health sector (19d), together with its homogeneity of degree 1 implies

$$\frac{\eta(h_e\tilde{e}+h_m\tilde{m})}{\omega\tilde{l}}=h_e\left(\frac{1-\tau_w}{1-s}\right)$$

and hence

$$\tilde{e}<\left(\frac{1-\tau_w}{1-s}\right)\frac{\omega\tilde{l}}{\eta}<\frac{\omega\tilde{l}}{\eta} \tag{A.8}$$

Thus, (A.7) and (A.8) together imply

$$1<\tilde{l}+\frac{\tilde{l}}{\eta}+\tilde{e}<\tilde{l}+\frac{\tilde{l}}{\eta}+\frac{\omega\tilde{l}}{\eta}$$

and hence we conclude:

$$\tilde{l}>\frac{\eta}{1+\eta+\omega} \tag{A.9}$$

A.3 Dynamics of the relative capital stock

To obtain the dynamics of individual capital we linearize equation (24') around the steady-state $\tilde{K},\tilde{l},\tilde{k}_i,\tilde{l}_i$. This is given by

$$\dot{k}_i(t)=(1-\tau_w)\frac{\tilde{A}F_L}{\tilde{K}}\left[\left(\frac{1+\eta+\omega}{\eta}\right)(\tilde{k}_i-\upsilon_i)(l(t)-\tilde{l})+\left[\left(\tilde{l}\left(\frac{1+\eta+\omega}{\eta}\right)-1\right)\left(k_i(t)-\tilde{k}_i\right)\right]\right] \tag{A.10}$$

For notational convenience, we let

$$(1-\tau_w)\frac{\tilde{A}F_L(\tilde{K},\tilde{L})}{\tilde{K}}\left[\tilde{l}\left(\frac{1+\eta+\omega}{\eta}\right)-1\right]=\pi$$

and rewriting equation (25) as

$$\upsilon_i=\frac{(1-\tilde{k}_i)\eta}{\tilde{l}\left(1+\eta+\omega\right)}+\tilde{k}_i$$

enables us to express (A.10) in the more compact form

$$\dot{k}_i(t)=\pi\left(k_i(t)-\tilde{k}_i\right)+(1-\tau_w)\frac{\tilde{A}F_L}{\tilde{K}}\frac{\left(\tilde{k}_i-1\right)}{\tilde{l}}\left(l(t)-\tilde{l}\right) \tag{A.10'}$$

The stable solution to this equation is

$$k_i(t)-1=\left(\tilde{k}_i-1\right)\left[1+(1-\tau_w)\frac{\tilde{A}F_L}{\tilde{K}}\int_t^\infty\left(1-\frac{l(\tau)}{\tilde{l}}\right)e^{-\pi(\tau-t)}d\tau\right] \tag{A.11}$$

Setting $t=0$ in (A.10) and noting that $k_{i.0}$ is given, we obtain

$$k_{i,0}-1=\left(\tilde{k}_i-1\right)\left[1+(1-\tau_w)\frac{\tilde{A}F_L}{\tilde{K}}\int_0^\infty\left(1-\frac{l(\tau)}{\tilde{l}}\right)e^{-\pi\tau}d\tau\right] \tag{A.12}$$

Thus, having determined $\tilde{K},\tilde{L}$, and the time path for $l(t)$ from (21), equation (A.12) determines $\tilde{k}_i$, and knowing $\tilde{k}_i$, (A.11) in turn determines the entire time path for $k_i(t)$.

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Figure 1 : Dynamics of Infections and Productivity

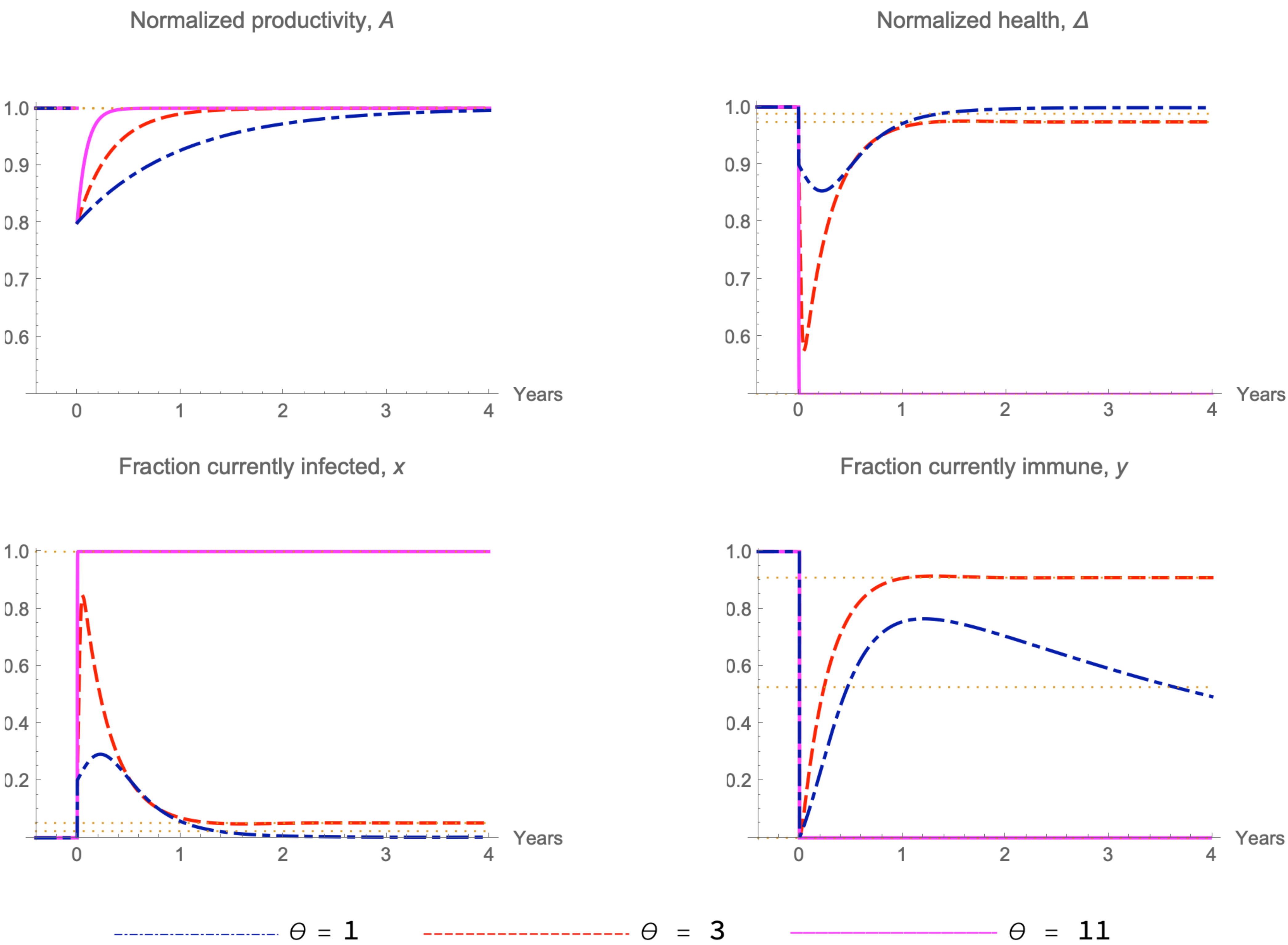

Figure 2 : Aggregate Dynamics

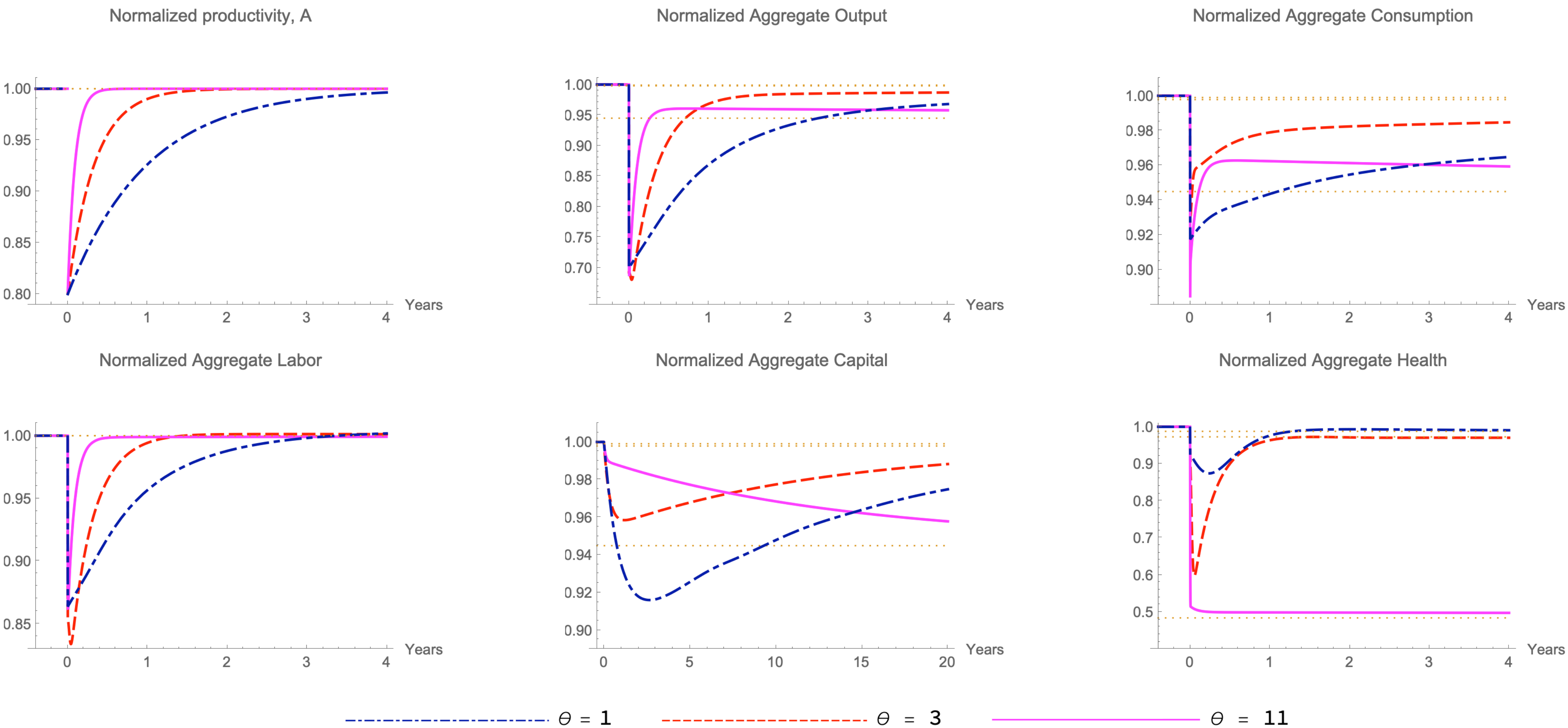

Figure 3 : Distributional Dynamics

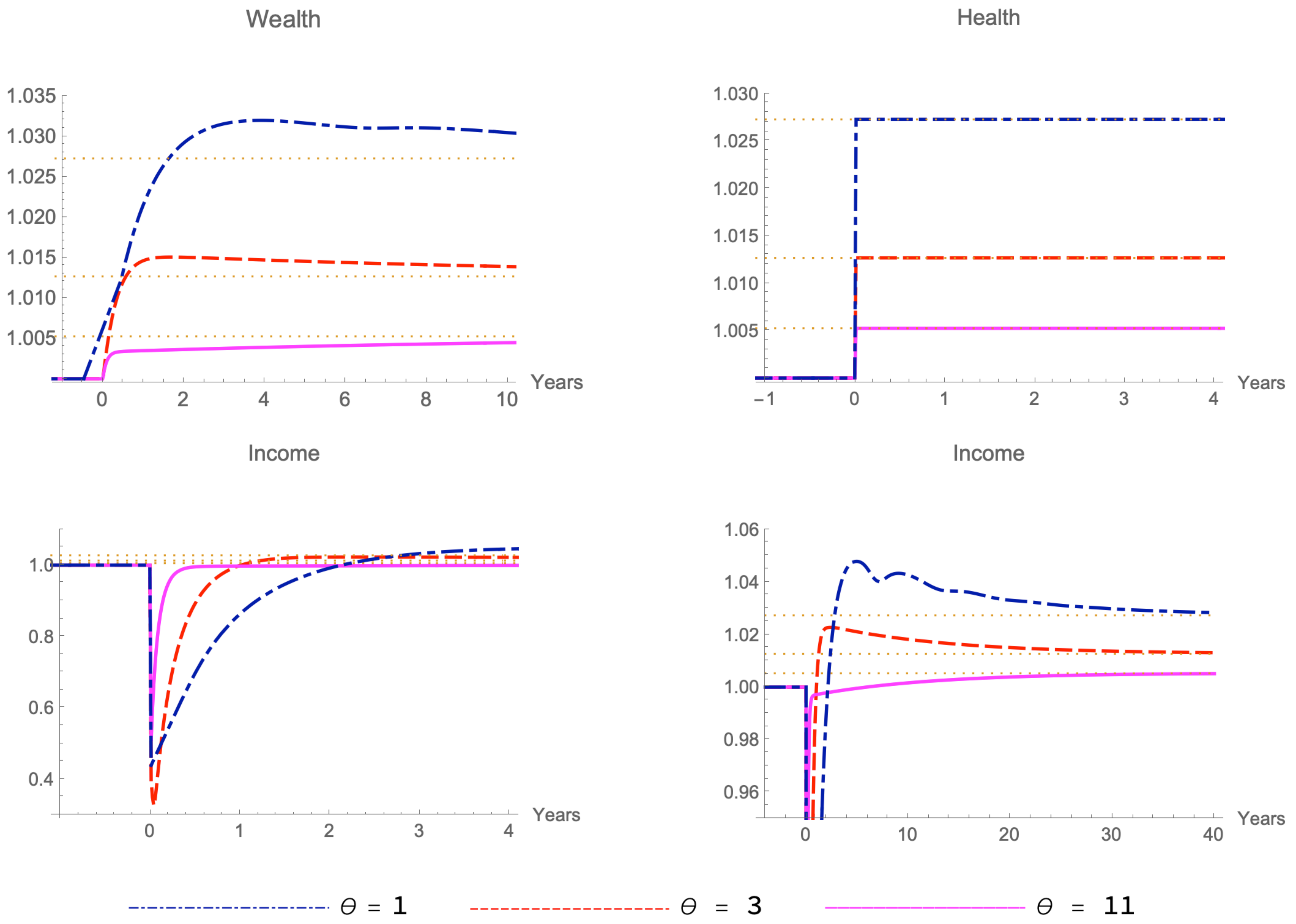

Figure 4 : Robustness to Duration of Immunity